7 Applying Global Flexibility

Rather than a reiteration of known results about the properties of common locally flexible functional specifications, the goal of the overview given in the first section of this chapter is to evaluate the suitability of these to serve as basis for a globally flexible or semi-non-parametric estimation with a constrained regular region in the sense outlined in the two preceding chapters. Since neither a simulation experiment nor an empirical application is conducted in the course of this study, the considerations remain speculative and are all exposed to the objection of arm-chair science. Correspondingly, this part of the study is rather cursory. Afterwards, an outlook will be given which identifies remaining weak spots of the presented approach and shows directions of further research, marking the way from a theoretical foundation of the global flexibility approach – which is hopefully provided by the study at hand – towards a broadly applicable methodical tool.

7.1 Selection of Functional Form

In a semi-nonparametric estimation with a constrained regular region, there is only one reason to incorporate theoretical requirements in a functional form: to save degrees of freedom and to relieve the accept-reject algorithm from the burden of accounting for them, respectively,[226] i.e. the reason to ease estimation.[227] If the statistically accepted regular region is large enough both from a hypothesis test and a forecast perspective, it is irrelevant if a theoretical requirement is accounted for by construction, by parametrical restriction or during the estimation process. Thus, the ability of a functional form to allow for the incorporation of possibly many properties of a well-behaved cost or profit function while remaining locally flexible, having dominated the whole functional form literature since the seventies, loses weight as a selection criterion. In turn, another critierion, having stayed in the background since then, gains importance: is the data generating process following a quadratic or a log-

[226] See section 6.2.3 above.

[227] Note that an implementation of e.g. homogeneity by parametrical restriction always applies globally. Thus, different regular regions imposed during the estimation process do not include properties thus enforced. This may have negative implications on estimablity if the respective property is not supported by the data outside the regular region. However, there is no way to find this out but an estimation where the respective property is implemented not before the estimation process so that it can be included in variations of the regular region.

linear course? How far does the approximating function behave like the true structure in a purely statistical sense? How well does the function actually approximate the data, theoretical properties left aside? This revival of the old question of an adequate specification leaves more room for choice between functional forms, since the ability to incorporate theory has led to very few functional forms – mostly not more than one – which were considered state of the art and thus acceptable as estimation functions. Hence, the following discussion of a small selection of functional forms has likewise to account for recent examples and specifications that are outdated in the context of second order flexible functional forms for their inferior ability to incorporate theoretical properties. This is not to imply that the extensive literature on regularity of second order functional forms is worthless for the purpose of this study – the opposite is the case: for the gobal flexibility account advocated here, it is of essential significance to draw upon knowledge of functional forms that allow a maximum of incorporated regularity while remaining fully flexible.

All functional forms discussed below in a historical order can be found in the systematics diagram in section 5.2.2 above. The choice of the functional forms presented here is somewhat arbitrary, but tries to emphasize both forms which are or were broadly used in applied studies, and/or can be seen as pathbreaking methodical progress. For the notation, please refer to section 5.2.1 above.

7.1.1 Generalized Leontief

The pioneering Generalized Leontief Cost Function, leading off the extensive literature on second order flexible functional forms motivated by the endeavour to make the progresses of duality theory empirically utilizable,[228] was introduced as

$$C(\boldsymbol{w}, y) = \sum_{i=1}^{n} \beta_i w_i + y \sum_{i=1}^{n} \sum_{j=1}^{n} \varphi_{ij} \left(w_i w_j \right)^{\frac{1}{2}} + \xi y^2 \sum_{i=1}^{n} v_i w_i$$

with $\varphi_{ij} = \varphi_{ji}$ and all v_i as predetermined constants to be selected by the researcher.[229] Since it does not treat input and output variables symmetrically, several multi-output generalizations are possible. Consider for example

[228] See section 1.1.2 and 1.1.3 above.

[229] See DIEWERT 1971: 497.

$$\Pi(\boldsymbol{p},\boldsymbol{w}) = \alpha_0 + \sum_{i=1}^{n} \beta_i w_i + \sum_{i=1}^{n} \gamma_i p_i + \frac{1}{2}\left(\sum_{i=1}^{n} v_i^p p_i\right)\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi_{ij}\left(w_i w_j\right)^{1/2} + \frac{1}{2}\left(\sum_{i=1}^{n} v_i^w w_i\right)\sum_{i=1}^{m}\sum_{j=1}^{m} \psi_{ij}\left(p_i p_j\right)^{1/2} + \sum_{i=1}^{n}\sum_{j=1}^{m} \theta_{ij} \ln w_i \ln p_j$$

which emphasizes the weighting of the second order terms with the respective other variables, here incorporated by an average of these, but simultaneously reveals the ugly asymmetry by means of the impossibility of formulating a reasonable weight for the mixed second order summand and the neglection of the last term of the original single-product cost function formulation – which is, again not exactly straightforward, a third order term, and it could be moreover argued that the original formulation does not include any proper second order terms because of the *y* weight. Another possibility, aesthetically more convincing, is

$$\Pi(\boldsymbol{p},\boldsymbol{w}) = \alpha_0 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi_{ij}\left(w_i w_j\right)^{1/2} + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \psi_{ij}\left(p_i p_j\right)^{1/2} + \sum_{i=1}^{n}\sum_{j=1}^{m} \theta_{ij}\left(w_i p_j\right)^{1/2}$$
$$= \alpha_0 + \frac{1}{2}\boldsymbol{w}^{1/2\prime}\Phi\boldsymbol{w}^{1/2} + \frac{1}{2}\boldsymbol{p}^{1/2\prime}\Psi\boldsymbol{p}^{1/2} + \boldsymbol{w}^{1/2\prime}\Theta\boldsymbol{p}^{1/2}$$

which assumes an indifference with respect to the question of whether a variable is an input or output variable. The β_i and γ_i terms can be dropped because they are equal to the φ_{ii} and ψ_{ii} terms, but this implies that the resulting functional form is no longer second order flexible by the number-of-parameters criterion. The Generalized Leontief is linearly homogeneous in prices by construction, but curvature and monotonicity can either be implemented locally only,[230] or, if restricted for globally, the second order flexibility property is lost.[231]

Similar to the multi-output generalization, the generalization for higher order flexiblity is not straightforward. One solution is the Asymptotically Ideal Production Model (AIM), of which the Generalized Leontief is the special case where the order of expansion equals one.[232] Another approach would be to continue the construction principle guaranteeing homogeneity with microfunctions including more parameters for higher order flexibility. In this case, the microfunctions assume the form

[230] See sections 6.1.1, 6.1.2, and 6.1.3 above.

[231] See LAU 1986: 1531-1539.

[232] See section 7.1.5 below.

$$\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\xi_{ijk}\left(w_i w_j w_k\right)^{1/3}$$

for third order terms.[233] Symmetry of all Hessians by Young's theorem implies that $\xi_{ijk} = \xi_{jik} = \xi_{ikj} = \xi_{kij} = \xi_{kji} = \xi_{jki}$ does not restrict the generality of the function, and is hence assumed.[234]

For terms of order t, the microfunctions can be written as

$$\frac{1}{t!}\sum_{i\in I^t}\vartheta_i\prod_{j=1}^{t}w_{i_j}^{1/t} \quad ,$$

where i denotes the multi-index over which summation is performed, and I^t is the set of t-tuples $I^t = \{i_1, i_2, \ldots, i_t\}$ with $i_j \in \{1,\ldots,n\}$ and $i_1 \le i_2 \le \ldots \le i_t$. For a comparison of the expansion and the higher order flexibility approach to increasing the depth of parameterization of locally flexible functional forms please refer to section 5.1.4.

Methods of incorporating curvature and monotonicity are uninteresting for a semi-nonparametric estimation based on the Generalized Leontief since they either work merely locally – as opposed to the requirement of implementing them for a regular region –, or, if global, destroy the flexibility property of the Generalized Leontief. This means that, in order to establish a functional form that satisfies the requirements of a globally flexible estimation, a Generalized Leontief which is linearly homogeneous by construction and otherwise unrestricted is perfectly suitable. This is so because the Generalized Leontief entertains only theoretical properties, namely homogeneity, which leave its capability unaffected to depict any consistent behavior without a priori restrictions not implied by economic theory – i.e. because it is locally flexible.

7.1.2 Trancendental Logarithmic (Translog)

The locally flexible functional form following the Generalized Leontief is the Trancendental Logarithmic or Translog.[235] The microfunctions of the Generalized Quadratic are of the form $f_i(q_i) = \ln q_i$, and the left hand variable, i.e. cost or profit, is logarithmized, too:

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233 In contrast to the second order case, a simplifying matrix notation is not available here.

234 See section 5.1.4 and 5.2.1 above.

235 See CHRISTENSEN/JORGENSON/LAU 1973: 256.

$$\begin{aligned}\ln\Pi(\boldsymbol{p},\boldsymbol{w}) &= \alpha_0 + \sum_{i=1}^{n}\beta_i \ln w_i + \sum_{i=1}^{m}\gamma_i \ln p_i \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\varphi_{ij}\ln w_i \ln w_j + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\psi_{ij}\ln p_i \ln p_j + \sum_{i=1}^{n}\sum_{j=1}^{m}\theta_{ij}\ln w_i \ln p_j \\ &= \alpha_0 + \beta' \ln \boldsymbol{w} + \gamma' \ln \boldsymbol{p} + \frac{1}{2}\ln \boldsymbol{w}' \Phi \ln \boldsymbol{w} + \frac{1}{2}\ln \boldsymbol{p}' \Psi \ln \boldsymbol{p} + \ln \boldsymbol{w}' \Theta \ln \boldsymbol{p}\end{aligned}$$

where the ln operator denotes taking logarithms element-wise, and with Φ and Ψ being symmetric. Setting Φ, Ψ, and Θ to null matrices reveals that the Translog is a generalization of the Cobb-Douglas functional form, which was introduced in 1928,[236] and, apart from the Constant Elasticities of Substitution (CES) form,[237] which entertains the Cobb-Douglas as limiting case, dominated applied economics until the development of the Translog, and is still widely in use. The Translog is probably the best investigated second order flexible functional form, and surely the one with the most applications. In contrast to the Generalized Leontief, the third and higher order extension is straightforward and exhibits the form

$$\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\xi_{ijk}\ln w_i \ln w_j \ln w_k$$

for third order terms.[238] Again, symmetry of all Hessians by Young's theorem implies that $\xi_{ijk} = \xi_{jik} = \xi_{ikj} = \xi_{kij} = \xi_{kji} = \xi_{jki}$ does not restrict the generality of the function and is hence assumed.[239] Higher order terms assume the form

$$\frac{1}{t!}\sum_{i\in I^t}\vartheta_i \prod_{j=1}^{t}\ln w_{i_j} \ ,$$

where the notation is the same as with the Generalized Leontief in section 7.1.1 above.

The theoretical properties of the second order Translog are well-known:[240] it is easily restrictable for global homogeneity, correct curvature can be implemented only locally if local flexibility shall be preserved, and monotonicity, as with all other derivates of the Generalized Quadratic, is impossible to maintain globally without losing second order flexibility. For hig-

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236 See COBB/DOUGLAS 1928.

237 See ARROW/CHENERY/MINHAS/SOLOW 1961.

238 In contrast to the second order case, matrix notation neither simplifies presentation nor makes the way in which the parameters and variables interact more obvious, and is thus omitted.

239 See section 5.1.4 and 5.2.1 above.

240 See LAU 1986: 1530-1533, and sections 6.2.1 through 6.2.3 above.

her orders, homogeneity can be incorporated in analogy with the second order case without flexibility loss, as will be demonstrated with a third order unit cost function to save notation:

$$\ln c(\boldsymbol{w}) = \alpha_0 + \sum_{i=1}^{n} \beta_i \ln w_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi_{ij} \ln w_i \ln w_j + \frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \xi_{ijk} \ln w_i \ln w_j \ln w_k$$

is globally linearly homogeneous by Euler's theorem if

$$\sum_{i=1}^{n} \frac{\partial \ln c(\boldsymbol{w})}{\partial \ln w_i} = \sum_{i} \beta_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \varphi_{ij} \ln w_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \xi_{ijk} \ln w_j \ln w_k \equiv 1$$

Since both the second and the third summand contain variables that can assume arbitrary values, the equation can only be restricted to a definite value if they are both restricted to zero. This is the case if

$$\sum_{j=1}^{n} \varphi_{ij} = 0 \qquad \forall i = 1,\ldots,n$$

$$\sum_{k=1}^{n} \xi_{ijk} = 0 \qquad \forall i, j = 1,\ldots,n$$

To restrict the entire equation to unity, the remaining summand has to equal unity:

$$\sum \beta_i = 1.$$

The Translog is perfectly suitable as a functional form for a semi-nonparametric estimation aiming at global flexibilty because, if merely restricted for global homogeneity, it has only theoretical properties which leave its capability to depict any behavior without a priori restrictions not implied by economic theory unaffected, i.e. is locally flexible. As the examination of the Generalized Leontief above yielded, and as it will reveal below, the same can be said about other second order flexible functional forms and their extensions to higher degrees of flexibility.

But the Translog entertains two advantages over all other specifications: first, it is extremely convenient to estimate, and secondly, it is likely to be a particularly good specification for economic processes. In addition, the unambigious multiple-output generalization may be seen as an advantage. With regard to the first point, all parameters of the Translog are evaluated when estimated as a system of $n-1$ cost share equations and the logarithmic cost function itself, where the form of the share equations, here formulated for the third order unit cost function above, is agreeably simple:

$$s_i = \frac{w_i x_i}{c(\boldsymbol{w})} = \frac{\partial \ln c(\boldsymbol{w})}{\partial \ln w_i} = \beta_i + \sum_{j=1}^{n} \varphi_{ij} \ln w_j + \sum_{j=1}^{n} \sum_{k=1}^{n} \xi_{ijk} \ln w_j \ln w_k$$

Considering the appropriacy for depicting economic processes, the Translog can be shown to posess a comparably large region of theoretical consistency when restricted for consistency for only one point.[241] This can be interpreted as the desirable property of being relatively close to theoretically consistent behavior by construction, i.e. by the way the variables interact in this specification prior to any trial to rule out theoretically inconsistent behavior by parametrical restrictions. Empirical evidence supports this mathematical finding: throughout applied economic literature, a relative superiority of the Translog with respect to statistical fit can be reported, which already occurred with its predecessor, the Cobb-Douglas form. A recent example is Terrell’s study where he applies a Translog, Generalized Leontief, and Symmetric Generalized McFadden cost function to the classical Berndt and Wood data, utilizing exactly the technique which is appropriate for the global flexibility concept advocated in this study. This result is especially interesting since the Generalized Leontief constitutes the second order flexible case of the AIM which is today's state of the art, suggesting that Translog extensions to higher order could frequently outdo the AIM too. One is tempted to conclude that the natural logarithm is closer to the true data generating processes of economic decision making than other transformations. In any case, the Translog is a particularly promising candidate for an application in a globally flexible estimation.

The possible objection that the Translog is more restrictive than other second order flexible functional forms, since it exhibits constant share elasticities is based on a misconception: all alternative candidates have not more than one parameter per effect either, so that their identical restrictiveness is merely veiled by their "ability" – also being a constraint in fact – to produce varying elasticities. More flexibilty requires more parameters, as with any other functional form. In contrast, the iso-elasticity property must be seen as an advantage over other forms since it allows an immediate parameter interpretation. However, this advantage is lost with higher order specifications because the elasticities then depend on more than one parameter.

[241] See LAU 1986: 1538. Also see CAVES/CHRISTENSEN 1980.

7.1.3 Fourier

The first series expansion to be introduced as an economic functional form is the Fourier model.[242] Due to the extensive notation which is requisite to formulate a Fourier cost function, and anticipating that it is in any case an inappropriate choice to depict cost minimizing behavior, as will become obvious immediately, it shall be refrained from listing it here. As a series expansion, the Fourier model is the first specification which allows semi-nonparametric estimation methods and was shown to be capable of a globally flexible estimation.[243] In addition, the basic principles of the recent techniques of imposing consistency on functional forms during the estimation process, set out in section 6.2.3 above, have been developed with it.[244] Nevertheless, the Fourier model is not an ideal candidate for approximating economic behavior since it consists of sines and cosines, i.e. is a periodic function which is not exactly close to a well-behaved economic function. Thus, it is not surprising that it is difficult to implement regularity without thereby restricting the flexibility of the Fourier form in an unacceptable manner. In addition, the technical expenditures of estimating its parameters seem to be that high that Barnett, Geweke, and Wolfe recommend leaving application of the Fourier model to professional econometricians. Both problems are to some extent overcome with the AIM presented in section 7.1.5 below.

7.1.4 Symmetric Generalized McFadden

Since its introduction in 1987, any econometric study of demand or supply behavior relying on another specification is exposed to the question of why the Symmetric Generalized McFadden was not utilized: this functional form is commonly considered state of the art, questioned not until the presentation of the AIM and the global flexibility approach, respectively.[245] The single-product cost formulation of the Symmetric Generalized McFadden, a close relative of the Generalized Leontief treated in section 7.1.1 above, is

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[242] See GALLANT 1981.

[243] See GALLANT 1982, see section 7.2 below.

[244] See GALLANT/GOLUB 1984.

[245] See section 7.1.5 below.

$$C(\boldsymbol{w},y)=\sum_{i=1}^{n}\beta_i w_i+\frac{1}{2}y\left(\sum_{i=1}^{n}v_i w_i\right)^{-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\varphi_{ij}w_i w_j+y\sum_{i=1}^{n}\theta_i w_i+y^2\xi\sum_{i=1}^{n}v_i w_i$$
$$=\boldsymbol{\beta'w}+\frac{1}{2}y\frac{\boldsymbol{w'\Phi w}}{\boldsymbol{v'w}}+y\cdot\boldsymbol{\theta'w}+y^2\xi\boldsymbol{v'w}$$

with $\varphi_{ij}=\varphi_{ji}$ and all v_i as predetermined constants to be selected by the researcher.[246] Possible multi-output generalizations and higher order extensions are analogous to the Generalized Leontief presented in section 7.1.1 above. Like the Generalized Leontief, the Symmetric Generalized McFadden is linearly homogeneous in prices by construction, and monotonicity can either be implemented locally only or, if restricted for globally, the second order flexibility property is lost.[247] But there is one important difference which provides the reason for the common distinction of the Symmetric Generalized McFadden as state of the art: if it is restricted for correct curvature by Lau's technique using the Cholesky decomposition,[248] the constrained curvature property applies globally. Unfortunately, the second order flexibility property is in this case restricted to only one point.[249] This drawback, recently reported as empirically meaningful against the original expectations,[250] and the incapability of incorporating the global monotonicity property could well constitute the best possible result that can be obtained with regard to the project of a globally regular and locally flexible functional form, noting that Lau's incompatibility theorem states that this ideal is in any case impossible to reach with linear functional forms.

The results of this study qualify the progress implied by the invention of the Symmetric Generalized McFadden. First, noting that theoretical consistency requires all properties to apply in conjunction, the ability to incorporate one more regularity property, but still not all, is irrelevant from an epistemological perspective. Secondly, in an applied context, it is likely that also accounting for curvature increases forecast credibility, but this must not necessarily be the case, in particular as the local flexibility property is harmed by this restriction.[251] Thirdly, the Bayesian method of incorporating regularity eliminates the relative attractivity of the Generalized Symmetric McFadden since, on the one hand, with this method the expendi-

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[246] See DIEWERT/WALES 1987a.

[247] See section 6.1.3 above.

[248] See section 6.1.2 above.

[249] See DIEWERT/WALES 1987a: 54.

[250] See RYAN/MAH 1994.

[251] See section 6.2.4 above.

ture of restricting second order flexibility to one data point is way too high compared to the advantage of being globally restrictable on curvature and, on the other hand, Lau's technique does not yield a globally correct curvature for higher order generalizations of the Symmetric Generalized McFadden, because the structure of the Hessian becomes more complex when third and higher order effects are considered so that it is not applicable in any case. Nevertheless, it is possible that the Symmetric Generalized McFadden, not restricted for correct curvature to preserve full flexibility, turns out to be a good specification to utilize in a globally flexible estimation simply because of the way the variables interact with it, i.e. that it overdoes other forms like the Translog or the AIM in a statistical sense, because it is a specification that is closer to the true data generating process.

7.1.5 Asymptotically Ideal Production Model (AIM)

The decisive step towards a broad applicability of semi-nonparametric methods and the progress of Gallant's and Golub's technique to account for theoretical consistency, namely the imposition of a regular region using numerical techniques, consists of the introduction of the Asymptotically Ideal Production Model, the AIM. It is a behavioral function based on the multivariate version of the Müntz-Szatz series expansion.[252] The AIM(t), i.e. with a series expansion of order t,[253] formulated as a single-product, constant returns to scale cost function formulation as in the original article,[254] assumes the form

$$C(\boldsymbol{w}, y) = y \sum_{i \in I^t} \vartheta_i \prod_{j=1}^{2^t} w_{i_j}^{2^{-t}} ,$$

with the index notation as introduced in section 7.1.1 above. To gain the feeling of its construction, note that the AIM(1) reduces to the Generalized Leontief, presented in section 7.1.1 above, and that the AIM(2) constant returns to scale cost function for a single output and, with respect to the last formula, for three inputs, can be written as

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[252] See BARNETT/JONAS 1983.

[253] Note the difference between the order of expansion and the order of terms or local flexiblity considered above.

[254] See BARNETT/GEWEKE/WOLFE 1991.

$$
\begin{aligned}
C(\boldsymbol{w},y) &= y\sum_{i_1=1}^{n}\sum_{i_2=1}^{n}\sum_{i_3=1}^{n}\sum_{i_4=1}^{n}\vartheta_{\{i_1,i_2,i_3,i_4\}}\prod_{j=1}^{4} w_{\{i_1,i_2,i_3,i_4\}_j}^{1/4} \\
&= y\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\vartheta_{ijkl} w_i^{1/4} w_j^{1/4} w_k^{1/4} w_l^{1/4} \\
&= y\sum_{i=1}^{n}\beta_i w_i + y\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\varphi_{ij} w_i^{1/2} w_j^{1/2} + \underbrace{y\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}}_{\substack{\forall\, i,j,k,l \text{ where at least one}\\ \text{index differs from all others}}}\xi_{ijkl} w_i^{1/4} w_j^{1/4} w_k^{1/4} w_l^{1/4} \\
&= y(\beta_1 w_1 + \beta_2 w_2 + \beta_3 w_3 \\
&\quad +\varphi_{12} w_1^{1/2} w_2^{1/2} + \varphi_{13} w_1^{1/2} w_3^{1/2} + \varphi_{23} w_2^{1/2} w_3^{1/2} \\
&\quad +\xi_{1112} w_1^{3/4} w_2^{1/4} + \xi_{1113} w_1^{3/4} w_3^{1/4} + \xi_{1222} w_1^{1/4} w_2^{3/4} + \xi_{1333} w_1^{1/4} w_3^{3/4} + \xi_{2223} w_2^{3/4} w_3^{1/4} + \xi_{2333} w_2^{1/4} w_3^{3/4} \\
&\quad +\xi_{1123} w_1^{2/4} w_2^{1/4} w_3^{1/4} + \xi_{1223} w_1^{1/4} w_2^{2/4} w_3^{1/4} + \xi_{1233} w_1^{1/4} w_2^{1/4} w_3^{2/4})
\end{aligned}
$$

where the notation approaches step by step the conventions used in this study. Considering the indices of the last formulation reveals that the "own-effects" reduce to the respective lower order effects

$$
\begin{aligned}
\varphi_{11} w_1^{1/2} w_1^{1/2} &= \beta_1 w_1 \\
\xi_{1122} w_1^{1/4} w_1^{1/4} w_2^{1/4} w_2^{1/4} &= \varphi_{12} w_1^{1/2} w_2^{1/2}
\end{aligned}
$$

and can thus be omitted. Global homogeneity of degree one is guaranteed by the exponents summing up to unity by construction, and global concavity, as with the Generalized Leontief, can be enforced by restricting all parameters to non-negativity, thereby sacrificing local flexibility and disqualifying it for a globally flexible estimation.[255]

Terrell performs a simulation study to examine the performance of the AIM where he utilizes Bayesian techniques to create a regular region following the Gallant and Golub approach of accounting for consistency. It is not surprising that he encounters a superiority of the AIM(3) against the AIM(1) or Generalized Leontief and a second order Translog. A more important result is that the AIM(3) significantly overcomes the problem of the AIM(1) to depict strong complements. Furthermore, he finds strong evidence that non-negativity constraints in order to enforce global concavity render the AIM incapable of approximating most of the simulated data, whatever order of expansion is chosen.[256] It yet remains to be checked whether the superiority of the AIM still exists if compared to a Translog specification with the same number of parameters or, in a semi-nonparametrical estimation, with that depth of pa-

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[255] See section 6.3 above.

[256] See TERRELL 1995.

of parameterization the data is able to support. Barnett, Geweke, and Wolfe suspect that the AIM has a particularly large natural regular region,[257] but this can well be a consequence of the higher degree of flexibilty in general rather than a property of the AIM specification in particular as opposed to e.g. the Translog specification: possibly, many regularity violations encountered with second order flexible functional forms are a result of their limited flexibilty rather than actually caused by the data. These considerations are somehow contradicted by the findings of Jensen, who plots regular regions for the AIM using data generated with a CDE functional form and, in contrast to Terrell's study, additive errors. He reports the smaller natural regular regions the higher the order of expansion is, and he observes that statistical fit is sometimes better with lower order AIMs.[258] However, Jensen, following his project of determining the natural regular regions of the AIM, does not restrict for consistency, so that his recommendation to be careful with higher order AIMs in the presence of noisy data is based on a wrong premise: his estimation, if it is misinterpreted as a model for an empirical application rather than a test of how large the natural regular regions are, is exposed to the objection that it does not account for regularity violations caused by overfitting by imposing regularity over a region, which is adequate no matter whether one estimates generated data which is known to be consistent but noisy, or real world data of which nothing is known a priori.

7.2 Outlook

There are several open questions with the presented approach, most of which can only be answered empirically or by performing simulation experiments, respectively. At least one question, however, is a purely mathematical one: for the Fourier expansion and the AIM, it is already shown that they possess the global flexibility property in the sense that they asymptotically can reach any continuous function. This is, however, not a trivial matter where it suffices to refer to the possibility of adding infinitely many parameters. Instead, this property must be verified in four steps: first, a matrix norm must be found to measure the distance between the approximated function and the approximation function, i.e. to measure the approximation error. In the case of the Fourier model and the AIM, the Sobolev norm was used. Then, it must be checked whether the examined functional form is, secondly, continuous, and thirdly, dense with respect to the norm. Finally, it has to be shown that the norm is continuous with

[257] See BARNETT/GEWEKE/WOLFE 1991: 41.

[258] See JENSEN 1997.

respect to the estimation method. For the details of this verification procedure see Gallant.[259] This procedure has to be performed for any functional form prior to its use in a globally flexible estimation, that is, with regard to the recommendation of this study to try the Translog, in particular for the higher-degree flexible Translog specification.

With regard to an application of the global flexibility concept, there are four approaches which could be considered: first, although there exist two studies on the performance of the AIM, neither of these provides the information which is needed to evaluate its suitability for the global flexibility approach proposed in this study. Rather, an analysis has to be performed which, first, uses consistently generated data with an additive error, like Jensen's study but unlike Terrell's, because otherwise, important properties like the behavior with regard to overfitting cannot be observed. Secondly, regularity must be enforced for a sufficiently large region, like Terrell's study but unlike Jensen's, who discontinues his efforts at an early stage because of local inflexibility. Thirdly, a comparison of different depths of parameterization with regard only to statistical criteria has not been performed, i.e. ignoring the share of rejections caused by consistency violations in implementing the regular region or the amount of binding restrictions, respectively, because it is impossible to find out whether these are caused by an inconsistent data generation process or by overfitting the noise in the data and thus irrelevant: Superiority in the context of the global flexibility account of this study requires a superior statistical fit only.

Secondly, another fruitful direction of further research would be to conduct a simulation experiment similar to the one proposed above with a semi-nonparametric Translog, given that it turns out that the Translog is theoretically suitable for a globally flexible estimation according to the criteria sketched in the first paragraph of this section. Reasons for a possible superiority of the Translog over the AIM are presented in section 7.1.2 above.

Thirdly, there is another aspect of the studies on semi-nonparametric estimation of dual behavioral functions, i.e. the three studies on the AIM mentioned in section 7.1.5 above, which calls for a revision: in all these studies there is discrimination between AIM(1), AIM(2), and AIM(3), whereas mixed parameterizations are neglected. As the considerations in section 5.1.4 above imply, this is by no means requisite. Instead, a statistically optimal approximation of the data would allow for an inhomogeneous depth of parameterization which is the greater the more an estimation of the respective parameters increases overall statistical significance.

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[259] See GALLANT 1982.

Fourthly, a positive answer to one of the questions above would, of course, suggest an empirical application of either the AIM or the Translog model or both with the methods outlined here. A particularly attractive choice would be the well-known Bernd-Wood data, which was already used in a number of comparative analyses of flexible functional forms. Another desirable application would be the compound feed cost model proposed in the outlook of the first part of this study, so that the two parts of this study could finally be united into one model.